Teaching and Outreach

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It is no big secret that in math, we could do better by drawing talent from more of the population than we currently do. In my teaching and service, I have been involved in efforts aimed at changing this: to broaden and diversify the pipeline of those who go into into STEM fields, and to find and improve conditions that prevent the people who are there from leaking out.

Here I briefly describe some of these activities.

1 Outreach

Broadening the pipeline

Activities that get kids on campus, into the math department or around math people can help them consider a future doing what we do. I have been involved in a variety of these kind programs, including:

- with Krashen, I designed and ran UGA MathCamp http://torsor.github.io/mathcamp/, which brought local high-school students to the math department at the University of Georgia for one week of mathematical activities in the summer (four summers 2013,14,16, and 18). We hosted a diverse group: generally 50% female, 40% minority, mainly African American students;
- I was a STEAM panelist, at the 2019 High-school Girls Career Pathways Conference, S. Brunswick, NJ;
- I lectured at the high-school Summer Stem Academy http://bit.ly/1PPQzKR (for two summers);
- I volunteered at the King/Chavez/Parks College Day Visitation Program http://bit.ly/2cxTdL5, where 7th and 8th graders from Detroit visited U of Michigan for a day of activities to promote interest in STEM;
- as a grad student at UT Austin, I co-organized the Saturday Morning Math Group, https://www.ma.utexas.edu/users/smmg/, a program for high school students at UT.

Retention

Support for early career mathematicians

- I have helped start the new **Early Career Section** of The AMS Notices http://www.angelagibney.org/the-early-career/, a community service project for people in the early stages of their career and those who mentor them. I plan the issues, find and coordinate people who write the articles.
- I have co-organized a number of conferences and workshops, including:
  - 2018 Algebraic Geometry Northeast Sectional meeting, AGNES, (with Borisov, Buch, and Krashen);
  - 2016 Summer Workshop in Algebraic Geometry at UGA, (with Deopurkar, Kass, and Tarasca);
  - 2015 Bootcamp for the Alg Geom Institute in Salt Lake City, Utah (co-organized with Coskun, Lieblich and DeFernex) http://bit.ly/2cEuRjz. Funding from NSF and NSA came through UGA;
  - several years of GAGS, the Georgia Algebraic Geometry Symposium (2010, 12, 13, 14, 17);

- the 2004 MRC Program in Snowbird, Utah (co-organized with Abramovich, and McKernan);
- started the Graduate Student Mock AMS Conference at UGA, which helped train graduate students to give short talks as well as to provide a regular, community building activity.

- I have spoken at conferences and workshops for graduate students, recently:
  - in June 2018, (5 invited) lectures at the Geometry of Moduli Spaces of Curves Summer School, International Center for Theoretical Physics, Trieste, Italy;
  - in June 2017, (4 invited) lectures at GAeL, Géométrie Algébrique en Liberté, http://gael-math.org/aboutgael, in Bath, UK;
- I have been a Co-PI for NSF funded programs for grad student training:
  - RTG Grant (DMS-1344994) http://bit.ly/2cW8iES;
  - VIGRE Grant (DMS-0738586) http://www.angelagibney.org/at-uga/.

Support for women in math

- I have spoken at conferences for female grad students and postdocs including:
  - *Strength In Numbers: a graduate workshop in number theory and related areas*, at Queen's University, in Kingston, Ontario, May 2018;
  - *the Graduate workshop in Algebraic Geometry for Women Mathematics of Minority Genders*, at Harvard and MIT, in Boston, MASS, 2017;
  - *Women in Science Seminar* at the IAS Women's program in Princeton, NJ. 2007;
- I have co-organized conferences for women mathematicians, including:
  - In 2015, (with Linda Chen), a special session in Algebraic Geometry for the bi-annual AWM Research Symposium, held at University of Maryland;
  - In 2009, (with Diane Maclagan and Jessica Sidman), the Connections for Women Conference at MSRI for the Jumbo Session in Algebraic Geometry;
- I have participated in AWM Chapter activities, helping to start and run an AWM chapter at Rutgers in 2017, and from 2010 -2016, being faculty contact of an AWM Chapter at UGA;
- I was an Enhancing Diversity in Graduate Education mentor (EDGE) http://bit.ly/2cW7j7I.

2 Teaching

The Graduate Student Mock AMS Conference

I developed and ran the Graduate Student Mock AMS Conference, a program intended to train graduate students in the art of giving a successful twenty minute general audience research talk. This activity was run as a seminar, and participants earned three credit hours. First and second year students often gave talks where they presented a subject covered on one of their qualifying exams. More advanced students spoke about their research. It was an excellent community building and training exercise. I believe this would be an excellent addition to any graduate program.

Graduate Student Teaching Training Seminar

In response to feedback from the graduate students who felt their current summer teaching training was too long winded and ineffectual, I redesigned some aspects of the teaching training program. With the new regime, each student gave a lecture, videotaped by a partner, watched by the pair together. Each student wrote: (1) a syllabus on an undergraduate or graduate class of their choosing; (2) a collection of tests to go along with their syllabus; and (3) a job application teaching statement. Students were also required to read another person's syllabus, tests, and teaching statement, and give verbal feedback.

Algebraic Geometry VIGRE Research Groups (VRGs)

In an effort to train graduate students in how to carry out a research project, as part of the VIGRE training grant, we instituted regular graduate student research training groups, referred to as VRGs. I was involved with four VRGs, briefly described below.

1. During Spring of 2009, I ran a VRG in which we studied a family of level one conformal blocks divisors on the moduli space of stable n pointed curves of genus zero. Our observations have appeared in a paper that has been published in the International Mathematical Research Notices, which I describe in my research statement. We used numerical data obtained via Maucaley2 to formulate ideas about the family, and the proofs mainly involved linear algebra, accessible to the students.

2. In Spring 2011, I supervised the second semester of the VRG led by Alexeev in the fall. The students lectured on work they had started during the previous semester. Two publications came out of the project, including a paper in Inventiones.

3. During my third VRG, held in 2013, my goal was for the students get to know various moduli spaces. I met with each student individually (usually two or three times), to plan and practice talks they would give to the class.

4. During the AY 2014 -15 I held a VRG focused on an open problem in the theory of conformal blocks which could be investigated by way of Schubert calculus. The problem was to determine whether or not bundles for sl_3 of positive rank with level below the theta and critical levels, have nontrivial first Chern class. This result holds for sl_2, and is known not to be true for sl_4. So the sl_3 case is interesting as it would complete the picture.

   Students took the first semester to learn Schubert calculus and necessary background. In second semester, the students designed experiments using Maucaley2 which led them to postulate that the sl_3 picture is analogous to sl_2. A subset of students is currently trying to prove their conjecture.

Graduate topics classes

1. **Toric Varieties:** Toric varieties are those on which an algebraic torus acts with a dense orbit. They are particularly good examples to study when learning algebraic geometry and commutative algebra, as many of their properties are completely determined by combinatorial information, and examples are in abundance. Because of their usefulness in transforming deep general questions into tractable combinatorial problems, they have served as a testing ground for important conjectures and have played a central role in many modern research programs. In this class, we used Fulton's book on toric varieties as well as lecture notes from various authors, to learn the basics. Students each gave a talk on something related to toric varieties.

2. **Quantum Cohomology:** The quantum cohomology ring is a generalization of the ordinary cohomology ring of a closed manifold. The idea is that in the quantum cohomology ring, ordinary intersection numbers which count how subspaces intersect is replaced by quantum cup products, which counts curves. There

are many numerical applications to quantum cohomology, and in this class, our focus was on two of them: first, Kontsevich's formula for rational plane curves, and second, computing the Verlinde formula for type A conformal blocks, which is done using elementary quantum cohomology in the Grassmannian. The course was computational, full of examples, and very concrete.

3. **The moduli space of curves $\overline{\mathcal{M}}_{g,n}$:** Points in $\mathcal{M}_{g,n}$ correspond to isomorphism classes of smooth curves of genus g with n distinct marked points. Intuitively, smooth curves degenerate to singular ones, and to study families of curves, it pays to work with a proper space whose points include curves that have singularities. In this course we studied $\overline{\mathcal{M}}_{g,n}$, the moduli space of stable n-pointed curves of genus g. Moduli spaces of curves occupy a distinguished position in algebraic geometry: They give insight into the study of smooth curves and their degenerations, they have played a principal role as a prototype for moduli of higher dimensional varieties, and as special varieties, they have been one of the chief concrete, nontrivial settings where the nuanced theory of the minimal model program has been exhibited and explored. We touched on all of these features of $\overline{\mathcal{M}}_{g,n}$.

4. **Vector bundles of conformal blocks on the moduli space of curves:**

   The moduli spaces $\overline{\mathcal{M}}_{g,n}$, admit vector bundles of so-called *conformal blocks*, which are basic objects in rational conformal field theory. There is a canonical identification of these vector spaces with global sections of certain line bundles on moduli (stacks) of vector bundles on curves, a fact known for smooth curves for some time. Certain combinatorial aspects of the moduli space of curves reflect underlying geometric structures embodied by vector bundles of conformal blocks, and properties of these vector bundles have been discovered using the combinatorics of the moduli of curves. In these lectures, my goal was to feature this interplay, and to point out questions on each side which remain open.

I have given three (invited) graduate lecture series on Vector bundles of conformal blocks:

1. in 2018, I gave 5 lectures at the Geometry of Moduli Spaces of Curves Summer School, International Center for Theoretical Physics, Trieste, Italy http://indico.ictp.it/event/8319/
2. in 2017, I gave 4 lectures at GAeL, Géométrie Algébrique en Liberté, https://sites.google.com/outlook.com/gael/home, in Bath, UK;
3. in 2013, I gave 5 lectures at a summer school at Universita Sapienza, Rome, Italy

   For each of these summer school programs I provided written notes to the students. For a five lecture course:

   - In the first lecture I introduced the moduli space of curves, and motivated interest in the study of globally generated vector bundles. I defined nef divisors, and the kinds of questions one might ask about them, explaining how one can often reduce questions for curves of arbitrary genus to questions for $g = 0$.
   - In the second lecture I described fibers of the Verlinde bundles both representation theoretically, and in terms of G-bundles.
   - In the third lecture I showed Verlinde bundles, defined on $\overline{M}_{g,n}$ for all g and all n, are globally generated for $g = 0$.
   - Lehmann and Fulger defined the pliant cone $PL^k(X)$ to be the closure of the cone generated by products Chern classes of globally generated vector bundles (maybe for different bundles) with total codimension k. I explained the relevance of the Pliant cone, and what we know about elements in $PL^k(\overline{M}_{0,n})$ using Verlinde bundles.
   - In the final lecture I discussed geometric interpretations of Verlinde bundles.