Homework 3

Problem 1

Make a character table for $\mathbb{Z}/2 \times \mathbb{Z}/2$.

Problem 2

Let G be a finite, non-abelian simple group (ie A_n for $n \geq 5$). Prove that the only group homomorphism $\chi : G \to S^1$ is trivial.

Problem 3

1. Write out all representations for D_4 and Q_8.
2. Write out the corresponding character tables (as maps to S^1).

Theorem C

(Can be used, does not need to be proven.) Let G be a finite abelian group, and $m \in \mathbb{Z}$. Then for any $g \in G$, $g^m = 1$ if and only if $\chi(g) = 1$ for every $\chi \in \widehat{G}$ which is an mth power in $\widehat{G}$.

Theorem D

(Can be used, does not need to be proven.) Let G be a finite abelian group, and $m \in \mathbb{Z}$. Then for any $g \in G$, g is an mth power in G if and only if $\chi(g) = 1$ for every $\chi \in \widehat{G}$ such that $\chi^m = 1_{\widehat{G}}$.

Problem 4

Finding the characters of $(\mathbb{Z}/(m))^r$:

1. For any $k \in \mathbb{Z}/(m)$, let $\chi_k : \mathbb{Z}/(m) \to S^1$ be defined by $j \mapsto e^{2\pi ijk/m}$. Show that $\chi_0, \chi_1, \ldots, \chi_{m-1}$ are all the characters of $\mathbb{Z}/(m)$, and that $\chi_k \chi_l = \chi_{k+l}$.
2. For $r > 1$, and r-tuples $\overrightarrow{a}, \overrightarrow{b} \in (\mathbb{Z}/(m))^r$, define $\overrightarrow{a} \cdot \overrightarrow{b} = a_1 b_1 + \cdots + a_r b_r \in \mathbb{Z}/(m)$. Let $\chi_{\overrightarrow{k}} : (\mathbb{Z}/(m))^r \to S^1$ be defined by $\overrightarrow{j} \mapsto e^{2\pi i(\overrightarrow{j} \cdot \overrightarrow{k})/m}$. Show that $\chi_{\overrightarrow{k}}$ is a character of $(\mathbb{Z}/(m))^r$, and that $\chi_{\overrightarrow{k}} \chi_{\overrightarrow{l}} = \chi_{\overrightarrow{k} + \overrightarrow{l}}$.