Conformal Blocks Divisors on $\overline{\mathcal{M}}_{0,n}$ from $\mathfrak{sl}_2$

Angela Gibney

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This talk is about joint work with

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Valery Alexeev and David Swinarski

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and

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Boris Alexeev

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Of course such morphisms are defined by base point free divisors.

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Moreover, these divisors are defined geometrically, and so one should be able to understand the varieties X onto which they map.

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Finally, I will explain how this family illustrates a general method for using Conformal Blocks divisors on $\overline{\mathcal{M}}_{0,n}$ to find nef divisors in $\overline{\mathcal{M}}_g$ for positive genus g.

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Let us start with a little bit of notation.

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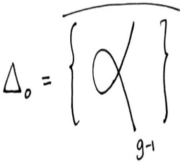

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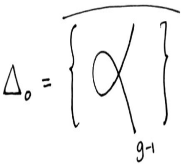

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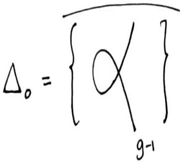

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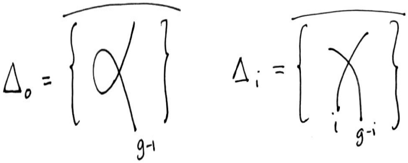

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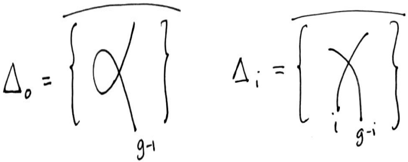

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However, we do know that $\overline{\mathrm{NE}}^1(\overline{\mathcal{M}}_3)$ is spanned by the classes δ_0 and δ_1 and the hyperelliptic locus $\mathcal{H}_3$.

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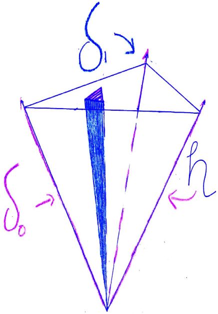

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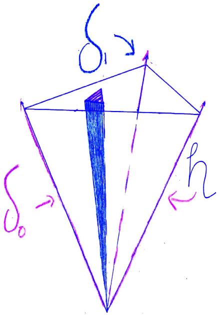

We'll look at a cross section, for which there is a partial chamber decomposition.

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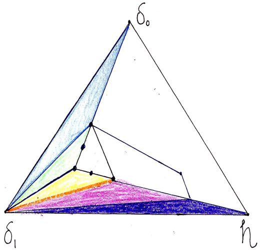

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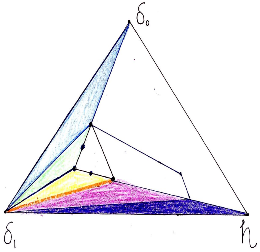

I will name the morphisms and (maps) as well as the images corresponding to some of these chambers.

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But there are extensions to various compactifications of $\mathcal{A}_g$.

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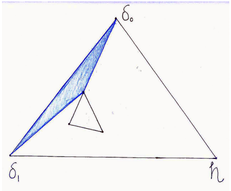

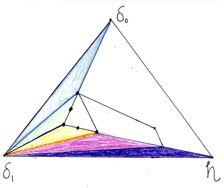

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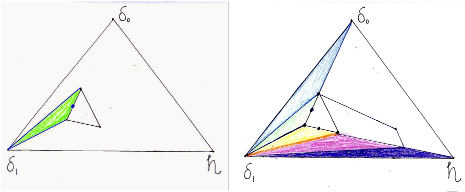

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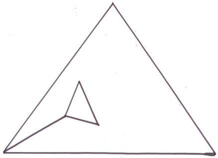

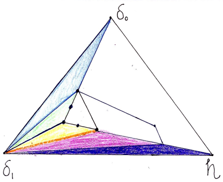

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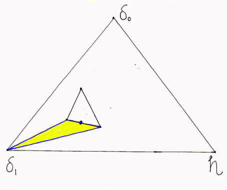

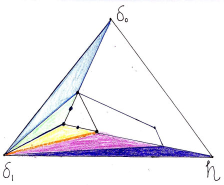

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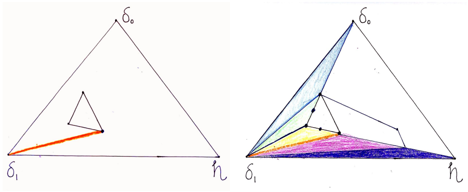

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$$\begin{tikzcd}
 & \overline{\mathcal{M}}_g \arrow[dl, "T"'] \arrow[dd] & & \\
\overline{\mathcal{M}}_g^{ps} \arrow[dr, "\psi"'] & & & (\overline{\mathcal{M}}_g^{ps})^+ = \overline{\mathcal{M}}_g^{hs} \arrow[dll, "\psi^+"] \\
 & \overline{\mathcal{M}}_g^{cs}. & &
\end{tikzcd}$$

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In this talk I'd like to convince you that to do this, we should use the divisors $\mathbb{D}^{\mathfrak{g}}_{\ell,\overline{\lambda}}$ on $\overline{\mathcal{M}}_{g,n}$ that come from the theory of Conformal Blocks.

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3. an n-tuple of dominant integral weights

$$\overline{\lambda} = \{\lambda_1, \ldots, \lambda_n\}$$

for $\mathfrak{g}$ of level ℓ.

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$$\mathrm{rk}(\mathbb{V}(\mathfrak{sl}_2, \ell, \overline{\lambda})) = (\frac{\ell+2}{2})^{g-1} \sum_{j=0}^{\ell} \frac{\prod_{i=1}^{n} sin(\frac{(i+1)(j+1)\pi}{\ell+2})}{(sin(\frac{(j+1)\pi}{\ell+2}))^{2g+n-2}}.$$

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Fakhruddin found an (amazing) recursive formula for their classes.

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$$c_1(\mathbb{V}(\mathfrak{g}, \ell, \overline{\lambda}))$$
$$= \frac{1}{(\ell + h^\vee)} \sum_{i=2}^{\lfloor \frac{n}{2} \rfloor} \epsilon_i \{ \sum_{\substack{A \subset [n] \\ |A|=i}} \{ \frac{r_{\overline{\lambda}}}{(n-1)(n-2)} \{(n-i)(n-i-1) \sum_{a \in A} c(\lambda_a)$$
$$+ i(i-1) \sum_{a' \in A^c} c(\lambda_{a'})\} - \{ \sum_{\mu \in P_\ell} c(\mu) \cdot r_{\overline{\lambda}_{A,\mu}} \cdot r_{\overline{\lambda}_{A^c,\mu^*}} \}\} \delta_A \}.$$

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Here $\epsilon_i = \frac{1}{2}$ if $i = \frac{n}{2}$ and 1 otherwise. $r_{\overline{\lambda}}$ are the ranks of corresponding CB vector bundles.

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the morphisms defined by them factor through contraction maps to certain moduli spaces of weighted stable curves, defined by Hassett.

For the rest of the talk I will focus on a different very simple family of CB-divisors defined on $\overline{\mathcal{M}}_{0,n}$.

On $\overline{\mathcal{M}}_{0,n}$, let

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So we put $n = 2(g+1)$ and consider the family

$$\{\mathbb{D}^{\mathfrak{sl}_2}_{\ell,\{1,\ldots,1\}} : 1 \leq \ell \leq g\}$$

of divisors on $\overline{\mathcal{M}}_{0,n}$.

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Features of the family:

I'll give a formula for the intersection of the divisors $\mathbb{D}^{\mathfrak{sl}_2}_{\ell,\{1,\dots,1\}}$ with a collection of curves that forms a basis for the 1-cycles on $\overline{\mathcal{M}}_{0,n}$.

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As an application, we identify and study the morphisms defined by the divisors and more generally, give a technique for finding Nef divisors on $\overline{\mathcal{M}}_{2(g+1)}$.

The divisors

$$B_k = \sum_{|A|=k} \delta_A$$

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We use F-Curves.

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$$\{1, \dots, n\} = A \amalg B \amalg C \amalg D.$$

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We denote the curve

$$F_{a,b,c,d}.$$

The set of F-Curves

$$\{F_{1,1,i,n-2-i} : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1\}$$

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In particular, it is enough to know how to intersect the divisors with these curves.

Suppose that n is even and let

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The family of divisors

$$\{D^{\mathfrak{sl}_2}_{\ell,\{1,\ldots,1\}} : 1 \leq \ell \leq g\}$$

forms a basis for $\mathrm{Pic}(\widetilde{\mathcal{M}}_{0,n})$*.*

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$F_{1,1,1}$	1	0	0	0	0	0	0
$F_{1,1,2}$	0	32	0	0	0	0	0
$F_{1,1,3}$	1	0	55	0	0	0	0
$F_{1,1,4}$	0	32	0	40	0	0	0
$F_{1,1,5}$	1	0	63	0	19	0	0
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The matrix has full rank. In the first and last two columns, many of the curves intersect the CB divisor in degree zero.

Theorem

The divisors in the set

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We see this by looking at the curves they intersect in degree zero.

For $n = 2(g+1)$, since $\mathrm{SymPic}(\overline{\mathcal{M}}_{0,n})$ is g-dimensional, any family of $g-1$ linearly independent curves determines an extremal ray of the nef cone in this vector space.

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To prove extremality of our divisors, we use F-curves.

$$D^{\mathfrak{sl}_2}_{1,\{1,\dots,1\}} \cdot F_{a,b,c,d} = 0 \iff abcd \equiv 0 \bmod 2.$$

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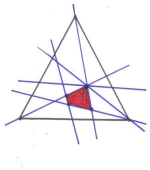

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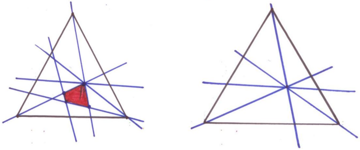

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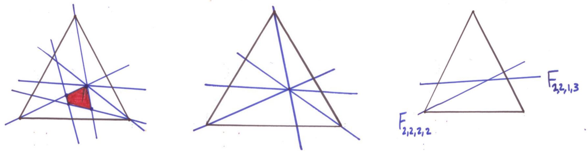

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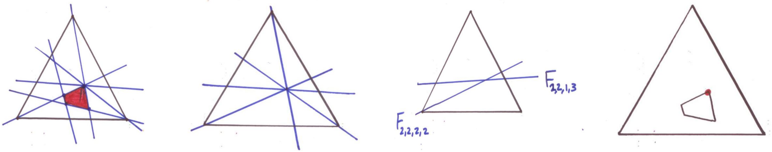

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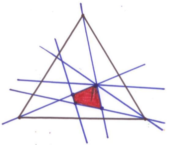

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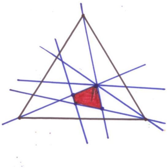

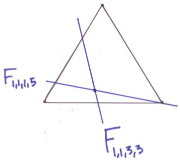

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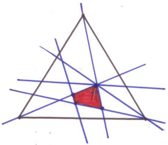

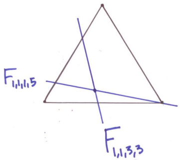

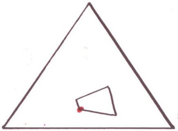

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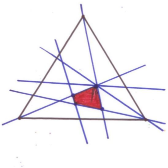

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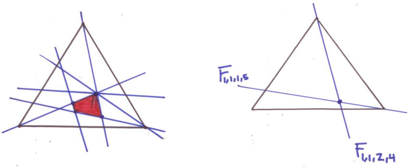

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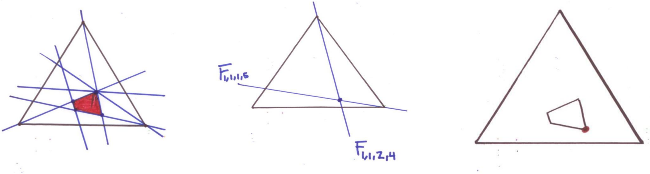

We next consider the morphisms defined by the extremal divisors and by divisors that lie on some faces spanned by them.

Theorem

The divisor $D^{\mathfrak{sl}_2}_{1,\{1,\ldots,1\}}$ defines the composition

$$\widetilde{\mathcal{M}}_{0,2g+2} \xrightarrow{h} \overline{\mathcal{M}}_g \xrightarrow{\bar{t}^{Sat}} \overline{\mathcal{A}}_g^{Sat}.$$

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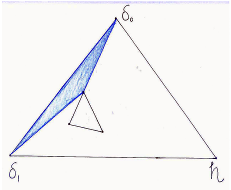

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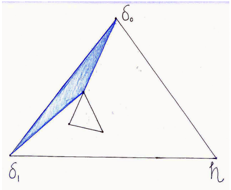

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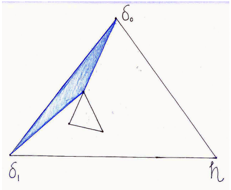

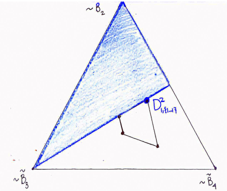

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$$D^{\mathfrak{sl}_2}_{1,\{1,\ldots,1\}} = h^*(\lambda).$$

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The divisor $D^{\mathfrak{sl}_2}_{2,\{1,\ldots,1\}}$ *defines the morphism*

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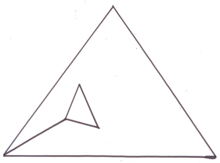

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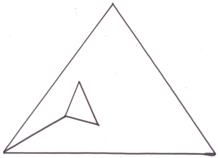

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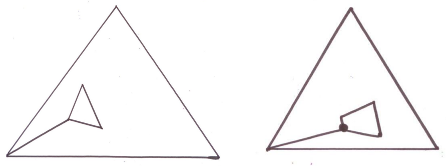

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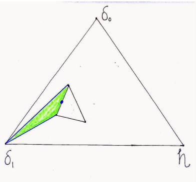

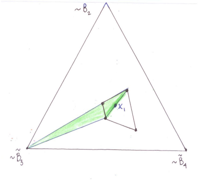

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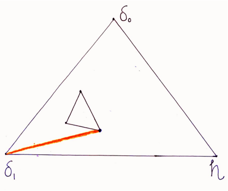

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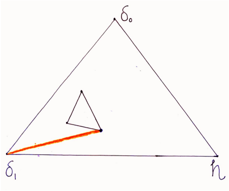

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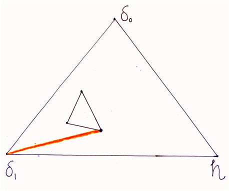

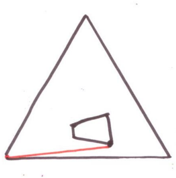

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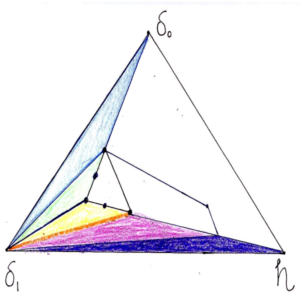

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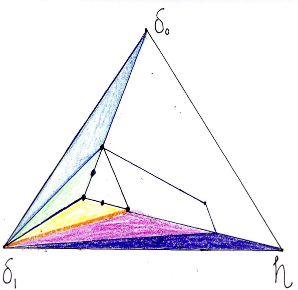

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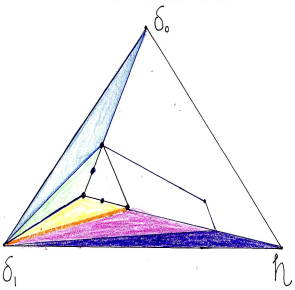

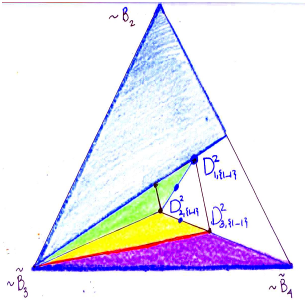

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This divisor was known to Alexeev-Swinarski and the blowup has been studied by Kiem and Moon (this is their notation).

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Moreover, presumably, as they are defined geometrically, we have a shot at identifying the morphisms entirely.

The End.

$\square$